# Erratum on shear stress and viscosity

In p. 100, **dynamic viscosity** μ (also called **absolute viscosity**) is defined as the the tangential force per unit area required to slide one plane with respect to another a unit distance apart at unit velocity.

This parameter appears by mistake as "η" in the Navier-Stokes equation (eqn. 3.3 in p. 102) and in all the derivations henceforth (eqns 3.5, 3.6, 3.7, 3.8, 3.9. 3.10, 3.12, 3.13, 3.14, and 3.15) in pp. 103-106.

When the shear stress tensor is introduced, in eqn. 3.17 the notation "μ" for viscosity is reinstated, but I inadvertently mixed up the two parameters when I derived the force (Eqn. 3.20 in p. 107). Note that, since η and μ are the same parameter, they cancel:

$$F_x = \mu \frac{\partial u_x}{\partial y} = \mu \frac{16h^2}{\eta \pi^3}\left(-\frac{dp}{dx}\right)\sum_{n=1,3,5\ldots}^{\infty}(-1)^{\frac{n-1}{2}}\left(1-\frac{\cosh\left(\frac{n\pi z}{2h}\right)}{\cosh\left(\frac{n\pi w}{2h}\right)}\right)\frac{\partial}{\partial y}\left[\frac{\cos\left(\frac{n\pi y}{2h}\right)}{n^3}\right]_{y=-h}$$

**Old equation 3.20**

$$F_x = \frac{16h^2}{\pi^3}\left(-\frac{dp}{dx}\right)\sum_{n=1,3,5\ldots}^{\infty}(-1)^{\frac{n-1}{2}}\left(1-\frac{\cosh\left(\frac{n\pi z}{2h}\right)}{\cosh\left(\frac{n\pi w}{2h}\right)}\right)\frac{\partial}{\partial y}\left[\frac{\cos\left(\frac{n\pi y}{2h}\right)}{n^3}\right]_{y=-h}$$

**New equation 3.20**

This equation can be rearranged the same way as in the textbook to yield:

$$F_x = \frac{8h}{\pi^2}\left(-\frac{dp}{dx}\right)\sum_{n=1,3,5\ldots}^{\infty}\left(1-\frac{\cosh\left(\frac{n\pi z}{2h}\right)}{\cosh\left(\frac{n\pi w}{2h}\right)}\right)\frac{1}{n^2}$$

**New equation 3.22**

This equation is a very good thinking exercise for your students. Ask them: "How can $F_x$ be independent of viscosity?". If we have cells in a microchannel and we run flow at a certain flow rate, and then we substitute the fluid by a more viscous fluid but keeping the flow rate constant, don't we increase the force exerted on the cells? If so, which is right, the formula or our intuition?

The answer lies in the (dp/dx) factor. If we go to any of the formulas of flow rate Q, say eqn. 3.8 for a rectangular channel, we see that Q, μ and (dp/dx) are interrelated:

$$Q = \frac{4}{3\mu} wh^3 \left(-\frac{dp}{dx}\right)\left[1 - \frac{192}{\pi^5}\frac{h}{w}\sum_{n=1,3,5\ldots}^{\infty}\frac{1}{n^5}\tanh\left(\frac{n\pi w}{2h}\right)\right]$$

**New equation 3.8**

Therefore, compared to a given fluid of viscosity μ, if we take another fluid of twice its viscosity, we will need twice the pressure drop (dp/dx) to maintain the same flow rate Q. Therefore, $F_x$ is dependent on viscosity indirectly via the pressure drop.